



PennState

CSE543

Introduction to Computer and  
Network Security

Module: Applied Cryptography

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- **Public Key cryptography**

- ▶ Each key pair consists of a public and private component:  $k^+$  (public key),  $k^-$  (private key)

$$D(E(p, k^+), k^-) = p$$

$$D(E(p, k^-), k^+) = p$$

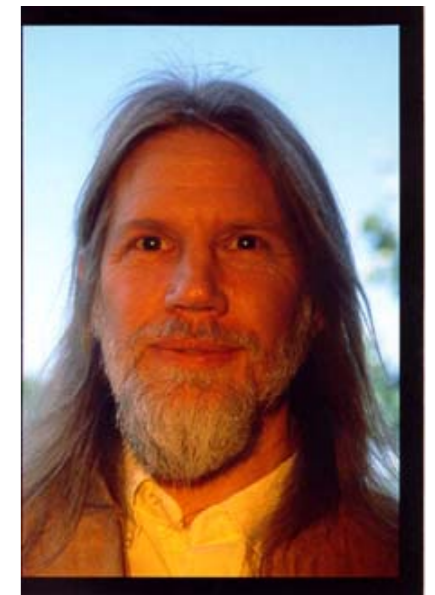
- **Public keys are distributed (typically) through public key certificates**

- ▶ Anyone can communicate secretly with you if they have your certificate
- ▶ E.g., SSL-based web commerce

- The DH paper really started the modern age of cryptography, and indirectly the security community
  - ▶ Negotiate a secret over an insecure media
  - ▶ E.g., “in the clear” (seems impossible)
  - ▶ **Idea:** participants exchange intractable puzzles that can be solved easily with additional information.

- **Mathematics are very deep**

- ▶ Working in multiplicative group  $G$
- ▶ Use the hardness of computing discrete logarithms in finite field to make secure



# Diffie-Hellman Protocol

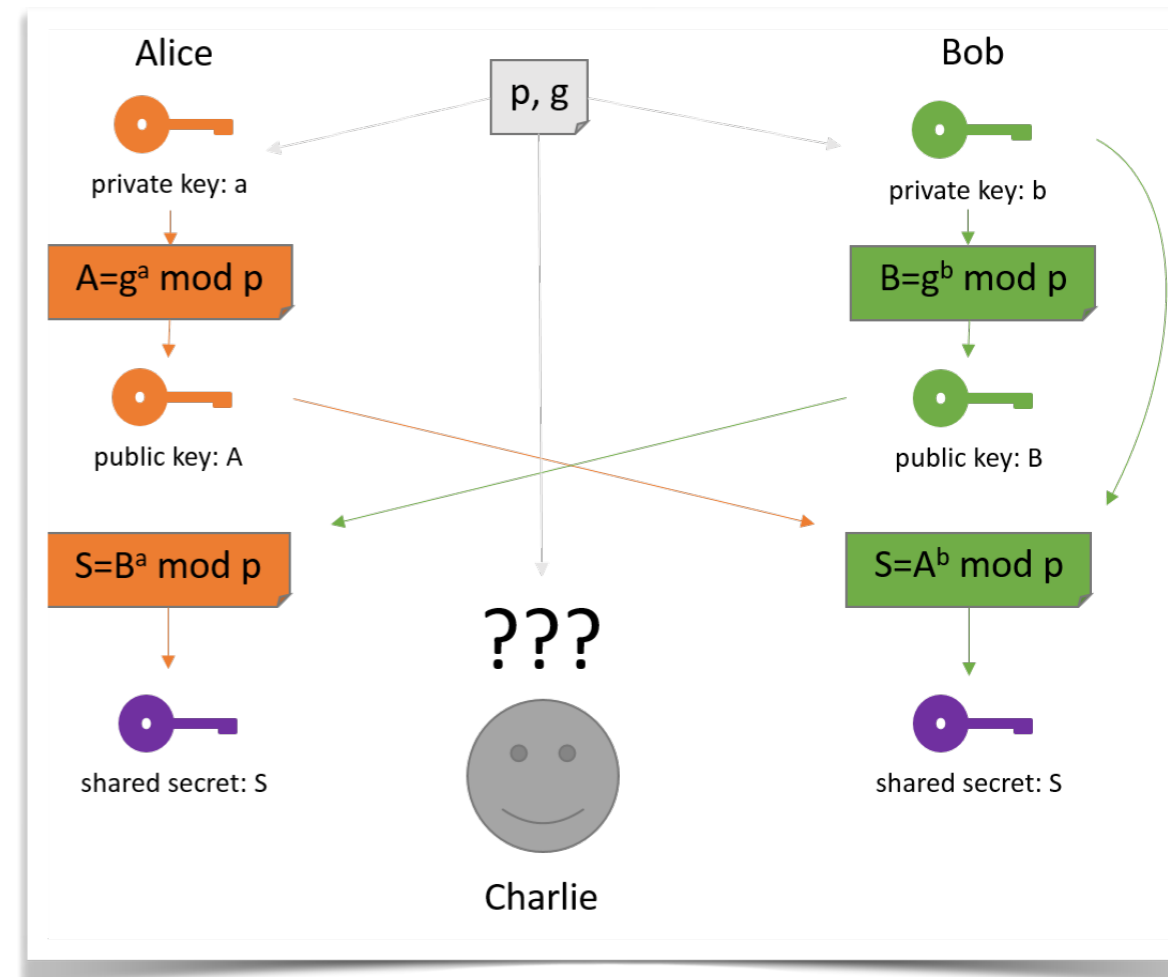
- For two participants  $p^1$  and  $p^2$
- Setup: We pick a prime number  $p$  and a base  $g (<p)$ 
  - ▶ This information is *public*
  - ▶ E.g.,  $p=13$ ,  $g=4$
- Step 1: Each principal picks a *private* value  $x (<p-1)$
- Step 2: Each principal generates and communicates a new value  $y$

$$y = g^{x1} \text{ mod } p$$

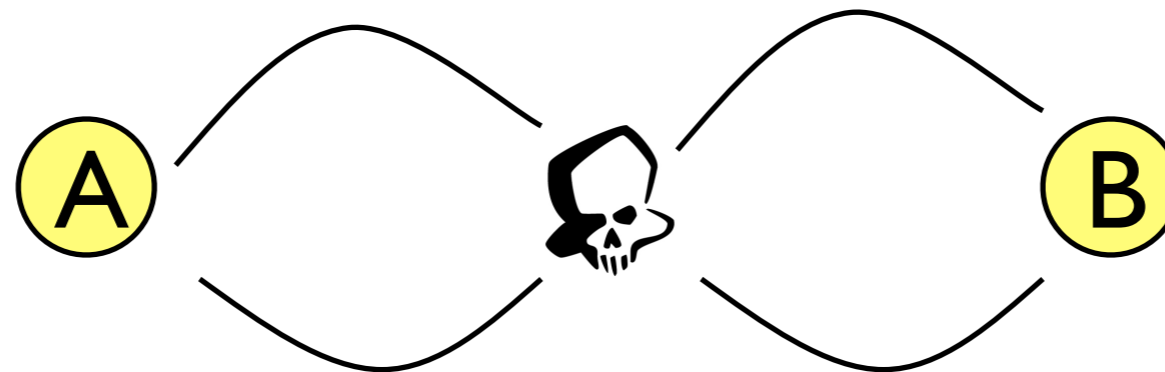
- Step 3: Each principal generates the secret shared key  $z$

$$z = y^{x2} \text{ mod } p = g^{x1x2} \text{ mod } p$$

Perform a neighbor exchange.



- This is key agreement, not authentication.
  - ▶ You really don't know anything about who you have exchanged keys with
  - ▶ The **man in the middle** ...



- ▶ Alice and Bob think they are talking **directly** to each other, but Mallory is actually performing two separate exchanges
- You need to have an authenticated DH exchange
  - ▶ The parties sign the exchanges (more or less)
  - ▶ See Schneier for a intuitive description

- **Key Distribution** is the process where we assign and transfer keys to a participant
  - ▶ Out of band (e.g., passwords, simple)
  - ▶ During authentication (e.g., Kerberos)
  - ▶ As part of communication (e.g., skip-encryption)
- **Key Agreement** is the process whereby two parties negotiate a key
  - ▶ 2 or more participants
- Typically, key distribution/agreement this occurs in conjunction with or after authentication.
  - ▶ However, many applications can pre-load keys



- A dominant public key algorithm
  - ▶ The algorithm itself is conceptually simple
  - ▶ Why it is secure is very deep (number theory)
  - ▶ Use properties of exponentiation modulo a product of large primes

"A Method for Obtaining Digital Signatures and Public Key Cryptosystems",  
Communications of the ACM,  
Feb., 1978, 21(2), pages  
120-126.



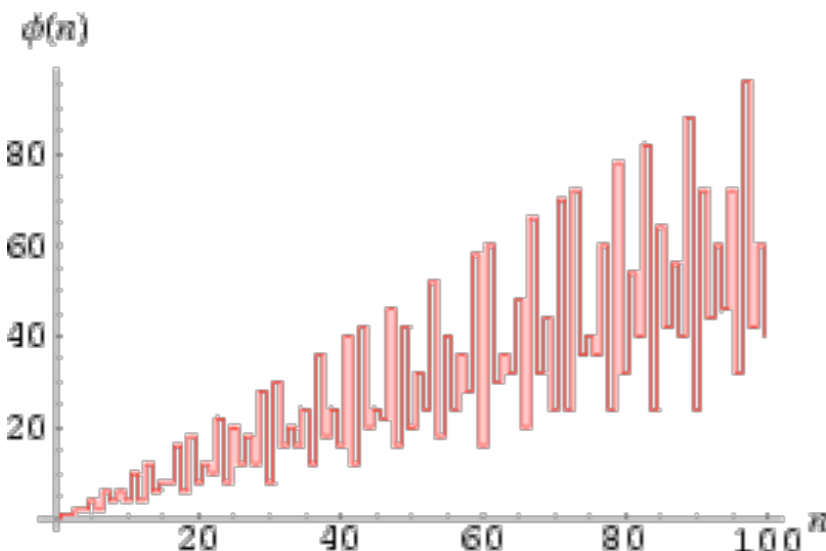
- $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
- $Z^+ = \{1, 2, 3, \dots\}$
- **prime vs. composite**
  - ▶ prime divide by only itself and 1 (has to be positive)
  - ▶ 0, 1 are not prime numbers
- **Prime factorization is unique**
  - ▶ fundamental theorem of arithmetic
  - ▶ Any integer greater than 1 can be written as a product of primes
    - $12 = 2 \times 2 \times 3$
    - if 1 were a prime  $12 = 1 \times 2 \times 2 \times 3 = 1 \times 1 \times 2 \times 2 \times 3$
- **If  $\text{GCD}(a,b) = 1$ , a and b are relatively prime**



- Any integer can be written like  $n = d \cdot q + r$ 
  - ▶  $d$  = divisor,  $q$  = quotient,  $r$  = result
- Two integers are congruent mod- $N$  if their difference is divisible by  $N$ 
  - ▶ if  $a = p \cdot N + r$ , and  $b = q \cdot N + r$ , then  $(a - b) = (p - q) \cdot N$
  - ▶  $a \equiv b \pmod{N}$  ( $a$  is congruent to  $b$  w.r.t. modulo  $n$ )
  - ▶  $a \pmod{N} = b \pmod{N}$
- Modular Multiplicative inverse
  - ▶ a modular multiplicative inverse of an integer  $a$  is an integer  $x$  such that the product  $ax$  is congruent to 1 w.r.t. modulus  $N$ .
  - ▶  $ax \equiv 1 \pmod{N}$
  - ▶  $3x \equiv 1 \pmod{10} ???$

# RSA Key Generation

- Pick two large primes  $p$  and  $q$
- Calculate  $n = pq$
- Pick  $e$  such that it is relatively prime to  $\phi(n) = (q-1)(p-1)$ 
  - “Euler’s Totient Function”
- $d \sim e^{-1} \pmod{\phi(n)}$  or  $de \pmod{\phi(n)} = 1$



1.  $p=3, q=11$
2.  $n = 3*11 = 33$
3.  $\phi(n) = (2*10) = 20$
4.  $e = 7 \mid \text{GCD}(20,7) = 1$
5. “Euclid’s Algorithm”  
 $d = 7^{-1} \pmod{20}$   
 $d \mid d7 \pmod{20} = 1$   
 $d = 3$

- Public key  $k^+$  is  $\{e,n\}$  and private key  $k^-$  is  $\{d,n\}$
- Encryption and Decryption
  - $E(k^+,P) : \text{ciphertext} = \text{plaintext}^e \bmod n$
  - $D(k^-,C) : \text{plaintext} = \text{ciphertext}^d \bmod n$
- Example
  - ▶ Public key (7,33), Private Key (3,33)
  - ▶ Data “4” (encoding of actual data)
  - ▶  $E(\{7,33\},4) = 4^7 \bmod 33 = 16384 \bmod 33 = 16$
  - ▶  $D(\{3,33\},16) = 16^3 \bmod 33 = 4096 \bmod 33 = 4$

- Encryption and Decryption

$$E(k^-, P) : \text{ciphertext} = \text{plaintext}^d \bmod n$$

$$D(k^+, C) : \text{plaintext} = \text{ciphertext}^e \bmod n$$

- E.g.,

- ▶  $E(\{3, 33\}, 4) = 4^3 \bmod 33 = 64 \bmod 33 = 31$

- ▶  $D(\{7, 33\}, 31) = 31^7 \bmod 33 = 27, 512, 614, 111 \bmod 33 = 4$

- Q: What is RSA's trapdoor function and trapdoor?

- Q: Why encrypt with private key?

- **Models physical signatures in digital world**
  - ▶ Association between private key and document
  - ▶ ... and indirectly identity and document.
  - ▶ Asserts that document is **authentic** and **non-reputable**
- **To sign a document**
  - ▶ Given document  $d$ , private key  $k^-$
  - ▶ Signature  $S(k^-, d) = E(k^-, h(d))$
- **Validation**
  - ▶ Given document  $d$ , signature  $S(k^-, d)$ , public key  $k^+$
  - ▶ Validate  $D(k^+, S(k^-, d)) = h(d)$



# Using Public Key Crypto

- Suppose you (Alice) want to send a document securely to another party (Bob)
  - You have each others' public keys
  - Obtained in some secure fashion (PKI, later)
- How do you send the document such that only Bob can read it?
- How do you send the document such that Bob knows it is from Alice?





- Survey by Dan Boneh

- ▶ <http://crypto.stanford.edu/~dabo/abstracts/RSAattack-survey.html>
- ▶ Real heavy math

- Results

- ▶ Fascinating attacks have been developed
- ▶ None devastating to RSA

- Cautions

- ▶ Improper use
- ▶ Secure implementation is non-trivial

- **Premise: Breaking RSA == Factoring Large Integers**
  - ▶ Factoring Large Integers is Hard
  - ▶  $N=pq$ ; if  $N$  is known, can we find  $p, q$ ?
- **Some Known (to cryptanalysts)**
  - ▶ If  $(p-1)(q-1)$  is product of prime factors less than some number  $B$
  - ▶  $N$  can be factored in time less than  $B^3$
- **Best Known Approach: General Number Field Sieve**
  - ▶ Significant early application by Arjen Lenstra

- **Fundamental tenet of cryptography**
  - ▶ Lots of smart people have tried but not (yet) figured out how to break RSA => RSA is secure
- **RSA Laboratories challenge (Mar 1991)**
  - ▶ Factor N into semiprimes (vary from 100 to 619 decimal digits).
  - ▶ Challenge ended in 2007
    - 16 of 54 listed numbers were factored
  - ▶ Current: up to 232 decimal digits factored
    - Using variations of “general number field sieve” algorithms

- **Common Modulus Misuse**

- ▶ Use the same  $N$  for all users
- ▶ Since all have a private key for same  $N$ 
  - Anyone can factor from their  $d$  and  $e$
  - Exposing any  $d$  is same as factoring  $N$

- **Blinding Misuse**

- ▶ Suppose adversary wants you to
  - Sign an arbitrary message  $M$
- ▶ You don't sign
- ▶ Adversary generates innocent  $M'$ 
  - Where  $M' = r^e M \pmod N$
  - Adversary can generate  $M$  signature from  $M'$  signature



- **Only use RSA (or any algorithm) in standard ways**

- **Secret key cryptography**

- ▶ Symmetric keys, where A single key (k) is used for E and D

- ▶  $D( E( p, k ), k ) = p$

- **All (intended) receivers have access to key**

- **Note: Management of keys determines who has access to encrypted data**

- ▶ E.g., password encrypted email

- **Also known as symmetric key cryptography**

- **Public key cryptography**

Each key pair consists of a public and private component:

$k^+$  (public key),  $k^-$  (private key)

$$D( E(p, k^+), k^- ) = p$$

$$D( E(p, k^-), k^+ ) = p$$

- **Public keys are distributed (typically) through public key certificates**

- Anyone can communicate secretly with you if they have your certificate

- E.g., SSL-based web commerce

- **Symmetric (shared) key systems**
  - ▶ Efficient (Many MB/sec throughput)
  - ▶ Difficult key management
    - Kerberos
    - Key agreement protocols
- **Asymmetric (public) key systems**
  - ▶ Slow algorithms (so far ...)
  - ▶ Easy (easier) key management
    - PKI - public key infrastructures
    - Webs of trust (PGP)





# Meet Alice and Bob ....

- *Alice* and *Bob* are the canonical players in the cryptographic world.
  - ▶ They represent the end points of some interaction
  - ▶ Used to illustrate/define a security protocol
- Other players occasionally join ...
  - ▶ *Trent* - trusted third party
  - ▶ *Mallory* - malicious entity
  - ▶ *Eve* - eavesdropper
  - ▶ *Ivan* - an issuer (of some object)



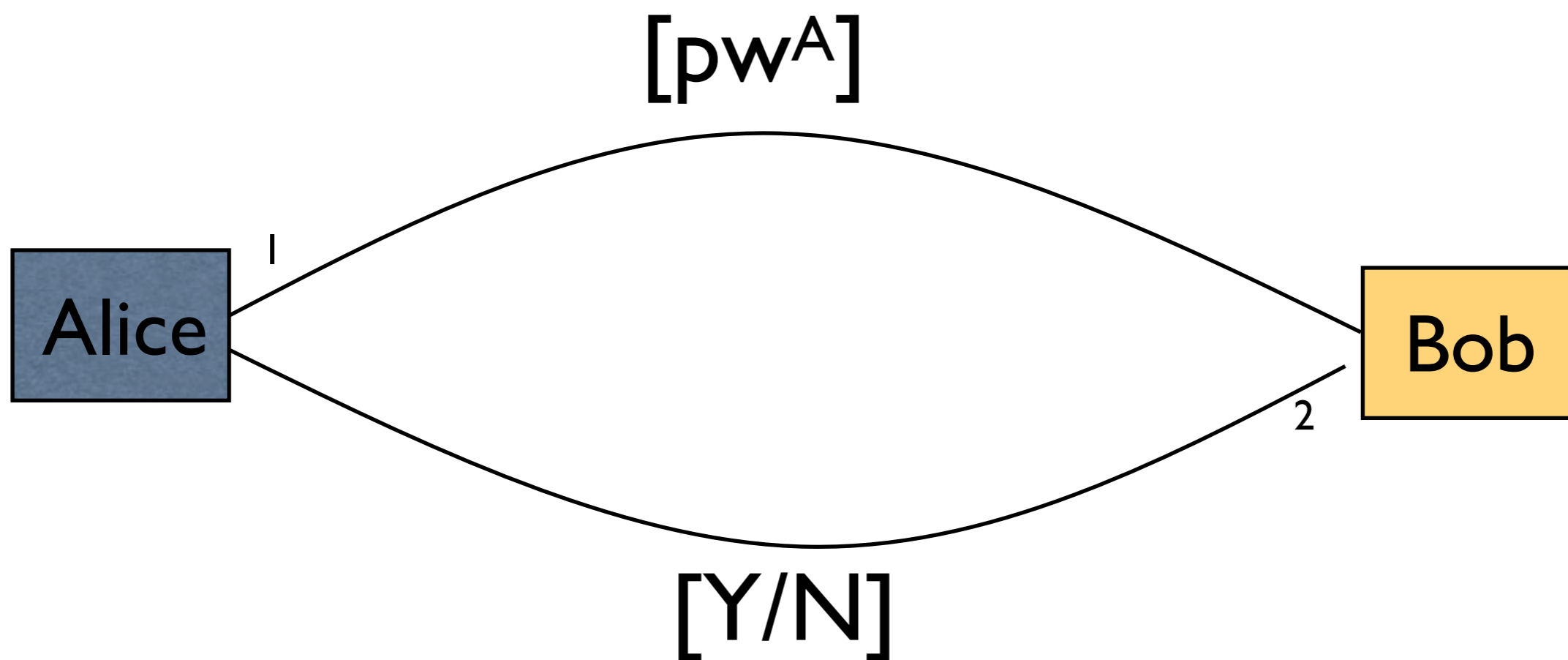
- You will generally see protocols defined in terms of exchanges containing some notation like
  - ▶ All players are identified by their first initial
    - E.g., Alice=A, Bob=B
  - ▶  $d$  is some data
  - ▶  $\text{pw}^A$  is the password for A
  - ▶  $k_{AB}$  is a symmetric key known to A and B
  - ▶  $K_A^+, K_A^-$  is a public/private key pair for entity A
  - ▶  $E(k, d)$  is encryption of data  $d$  with key  $k$
  - ▶  $H(d)$  is the hash of data  $d$
  - ▶  $S(K_A^-, d)$  is the signature (using A's private key) of data  $d$
  - ▶ “+” is used to refer to concatenation

- ... when communicating.
  - ▶ Ensure the *authenticity* of a user
  - ▶ Ensure the *integrity* of the data
    - Also called *data authenticity*
  - ▶ Keep data *confidential*
  - ▶ Guarantee *non-repudiation*



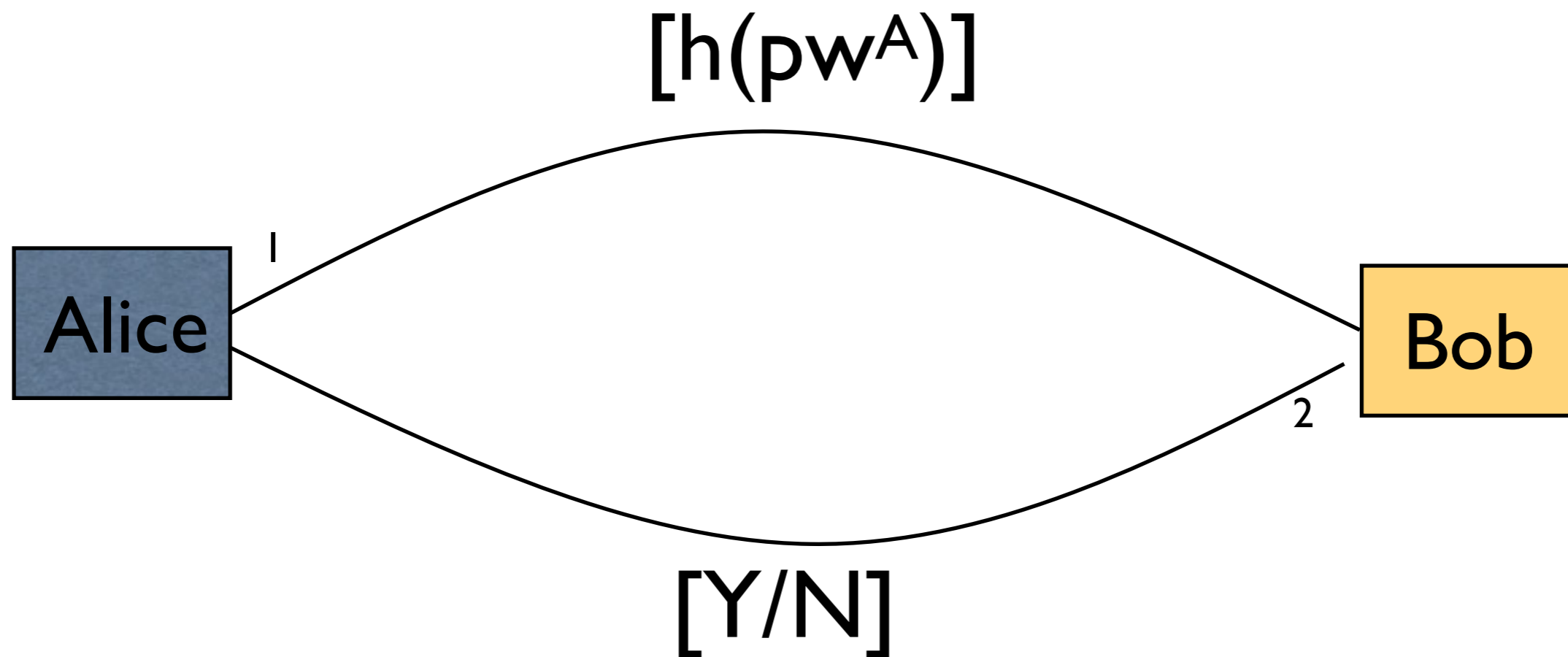
# Basic (User) Authentication

- Bob wants to authenticate Alice's identity
  - ▶ (is who she says she is)



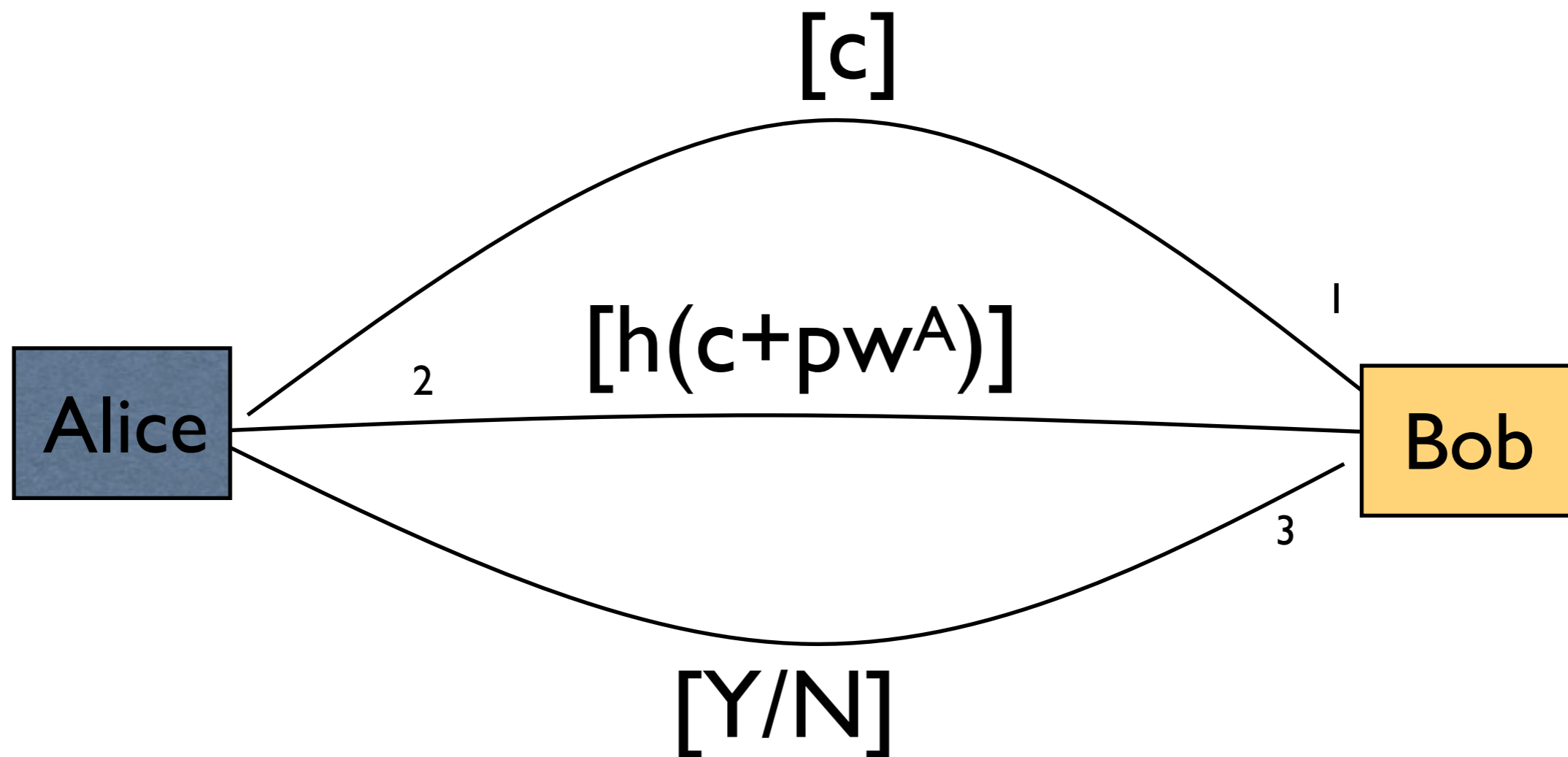
# Hash User Authentication

- Bob wants to authenticate Alice's identity
  - ▶ (is who she says she is)





- Bob wants to authenticate Alice's identity
  - ▶ (is who she says she is)





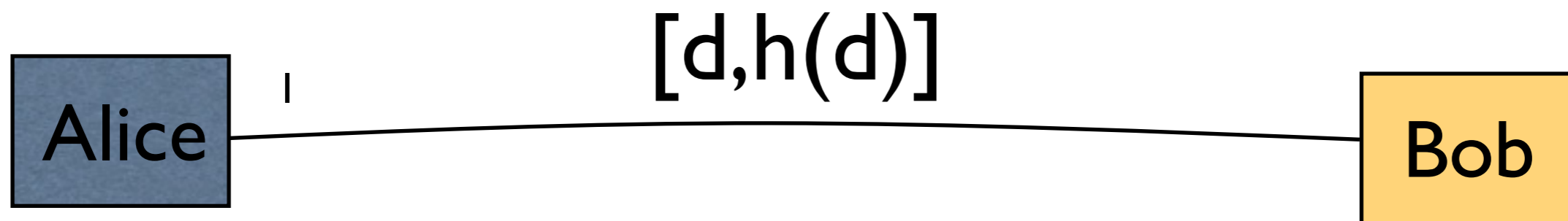
- User authentication proves a property about the communicating parties
  - ▶ E.g., I know a password
- Data integrity ensures that the data transmitted...
  - ▶ Can be verified to be from an authenticated user
  - ▶ Can be verified to determine whether it has been modified



- Now, lets talk about the latter, *data integrity*

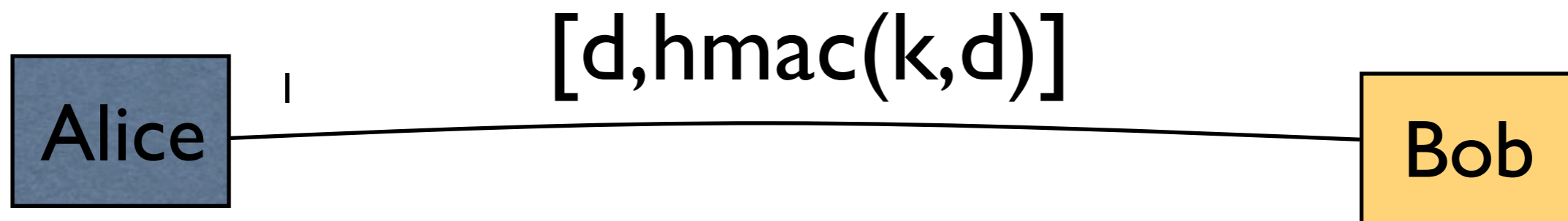
# Simple Data Integrity?

- Alice wants to ensure any modification of the data in flight is detectable by Bob (integrity)



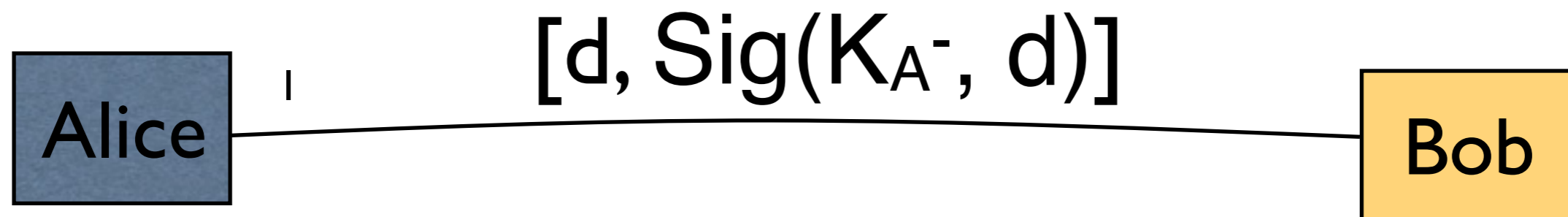
# HMAC Integrity

- Alice wants to ensure any modification of the data in flight is detectable by Bob (integrity)



# Signature Integrity

- Alice wants to ensure any modification of the data in flight is detectable by Bob (integrity)



# Data Integrity vs. Non-repudiation

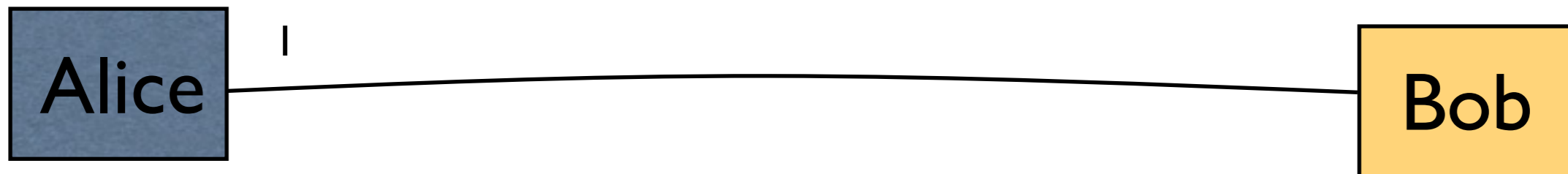
- If the integrity of the data is preserved, is it provably from that source?
  - ▶ HMAC integrity says what about non-repudiation?
  - ▶ Signature integrity says what about non-repudiation?



# Confidentiality

- Alice wants to ensure that the data is not exposed to anyone except the intended recipient (confidentiality)

$[E(k_{AB}, d), \text{hmac}(k_{AB}, d)]$



# Question

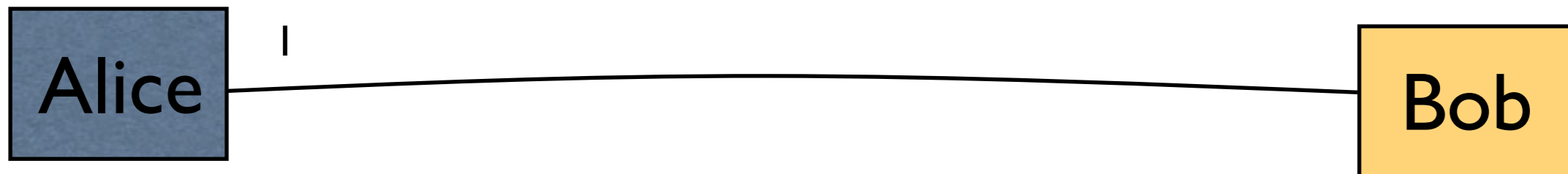
- If I already have an authenticated channel (e.g., the remote party's public key), why don't I simply make up a key and send it to them?



# Confidentiality

- Alice wants to ensure that the data is not exposed to anyone except the intended recipient (confidentiality)
- But, Alice and Bob have *never met*!!!!

$[E(k_x, d), \text{hmac}(k_x, d), E(K_B^+, k_x)]$



- Alice randomly selects key  $k_x$  to encrypt with

- How do we distribute a key in an untrusted network?
  - ▶ Diffie-Hellman
    - Beware of Man-in-the-Middle Attacks
  - ▶ Public key
    - Offline and via certificates (more later)
    - What about without certs
  - ▶ Symmetric key
    - Offline
    - How about online?

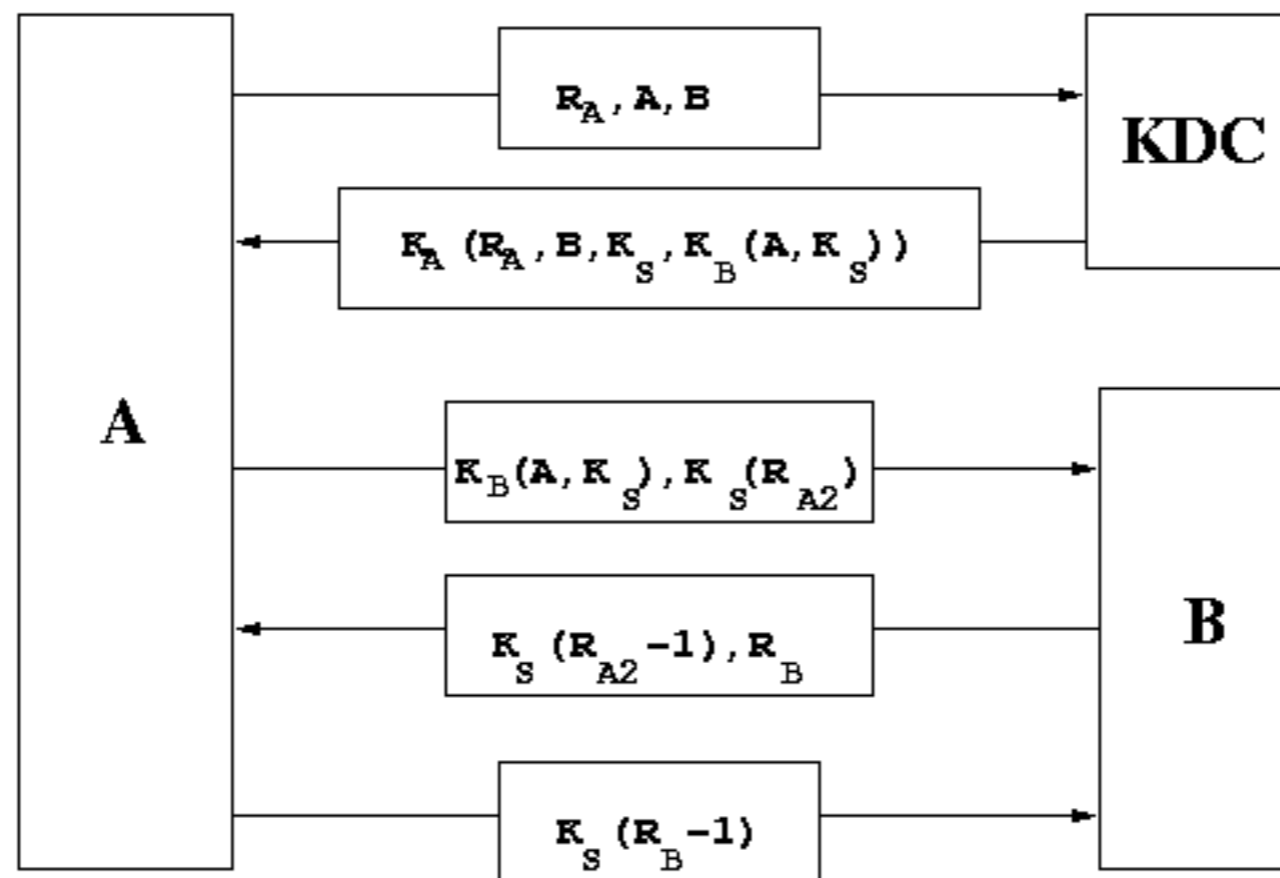


- **Goal**
  - ▶ Two parties want to communicate securely
- **Threat Model**
  - ▶ Network is untrusted
  - ▶ Other nodes may be untrusted
- **Requirements**
  - ▶ Mutual Authentication
  - ▶ Prove that only the appropriate parties hold secrets
- **Assumptions**
  - ▶ Trusted Third Party



# N-S Protocol

- For Symmetric Key Cryptosystems



**The Needham-Schroeder Authentication Protocol**

# N-S Protocol Detail



- **Message 1:**  $A \rightarrow S : A, B, N_A$  [N=R=nonce-random value]
  - ▶ A asks TTP S for a session key for A and B to use
- **Message 2:**  $S \rightarrow A : \{N_A, B, K_{AB}, \{K_{AB}, A\}_{BS}\}_{AS}$ 
  - ▶ S returns messages for A that includes the session key
  - ▶ And a message for A to give to B
- **Message 3:**  $A \rightarrow B : \{K_{AB}, A\}_{BS}$ 
  - ▶ A passes “ticket” on to B
- **Message 4:**  $B \rightarrow A : \{N_B\}_{AB}$ 
  - ▶ B asks A to demonstrate knowledge of  $K_{AB}$  through  $N_B$
- **Message 5:**  $A \rightarrow B : \{N_B^{-1}\}_{AB}$ 
  - ▶ A does!

Nonce



- Message a.1:  $A \rightarrow B : A, B, \{N_A, A\}_{PK_B}$ 
  - ▶ A initiates protocol with fresh value for B
- Message a.2:  $B \rightarrow A : B, A, \{N_A, N_B\}_{PK_A}$ 
  - ▶ B demonstrates knowledge of  $N_A$  and challenges A
- Message a.3:  $A \rightarrow B : A, B, \{N_B\}_{PK_B}$ 
  - ▶ A demonstrates knowledge of  $N_B$
- A and B are the only ones who can read  $N_A$  and  $N_B$

# A Protocol Story

- **Needham-Schroeder Public Key Protocol**
  - ▶ Defined in 1978
- **Assumed Correct**
  - ▶ Many years without a flaw being discovered
- **Proven Correct**
  - ▶ BAN Logic (early 1990s)
- **So, It's Correct, Right?**





# Gavin Lowe Attack



- An active intruder  $X$  participates...
- Message a.1:  $A \rightarrow X : A, X, \{N_A, A\}_{PKX}$
- Message b.1:  $X(A) \rightarrow B : A, B, \{N_A, A\}_{PKB}$ 
  - ▶  $X$  as  $A$  initiates protocol with fresh value for  $B$
- Message b.2:  $B \rightarrow X(A) : B, A, \{N_A, N_B\}_{PKA}$
- Message a.2:  $X \rightarrow A : X, A, \{N_A, N_B\}_{PKA}$ 
  - ▶  $X$  asks  $A$  to demonstrate knowledge of  $N_B$
- Message a.3:  $A \rightarrow X : A, X, \{N_B\}_{PKX}$ 
  - ▶  $A$  tells  $X$   $N_B$ ; thanks  $A$ !
- Message b.3:  $X(A) \rightarrow B : A, B, \{N_B\}_{PKB}$ 
  - ▶  $X$  completes the protocol as  $A$

# What Happened?

- What is the cause of this attack?



# What Happened?

- $X$  can get  $A$  to act as an “oracle” for nonces
  - ▶ Hey  $A$ , what’s the  $N_B$  in this message from any  $B$ ?
- $A$  assumes that any message encrypted for it is legit
  - ▶ Bad idea
- $X$  can enable multiple protocol executions to be interleaved
  - ▶ Should be part of the threat model



# The Fix

- It's Trivial (find it)
- Message a.1:  $A \rightarrow B : A, B, \{N_A, A\}_{PK_B}$ 
  - ▶ A initiates protocol with fresh value for B
- Message a.2:  $B \rightarrow A : B, A, \{N_A, N_B, B\}_{PK_A}$ 
  - ▶ B demonstrates knowledge of  $N_A$  and challenges A
- Message a.3:  $A \rightarrow B : A, B, \{N_B\}_{PK_B}$ 
  - ▶ A demonstrates knowledge of  $N_B$

- Protocol Analysis Took a Black Eye
  - ▶ BAN Logic Is Insufficient
  - ▶ BAN Logic Is Misleading
- Protocol Analysis Became a Hot Topic
  - ▶ Lowe's FDR
  - ▶ Meadow's NRL Analyzer
  - ▶ Millen's Interrogator
  - ▶ Rubin's Non-monotonic protocols
  - ▶ ....
- In the end, could find known flaws, but...
  - ▶ Attacker model is too complex

# Dolev-Yao Result



- **Strong attacker model**
  - ▶ Attacker intercepts every message
  - ▶ Attacker can cause operators to be applied at any time
    - Operators for modifying, generating any kind of message
  - ▶ Attacker can apply any operator except other's decryption
- **Theoretical Results**
  - ▶ Polynomial Time for One Session
  - ▶ Undecidable for Multiple Sessions
  - ▶ *Moral: Protocol Validation is Difficult Because Attacker Can Exploit Interactions of Multiple Sessions*

- The reality of the security is that 90% of the frequently used protocols use some variant of these constructs.
  - ▶ So, get to know them ... they are your friends
  - ▶ We will see them (and a few more) over the semester

- They also apply to systems construction

- ▶ Protocols need not necessarily be online
- ▶ Think about how you would use these constructs to secure files on a disk drive (integrity, authenticity, confidentiality)
- ▶ We will add some other tools, but these are the basics

